STEADY HEAT-CONDUCTION PROBLEM FOR A DOUBLE CYLINDER

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An analytical solution of the problem stated in the title is obtained for various conditions at the ends of the cylinder. The method of asymptotic solution of boundary-value problems for elliptic equations in thin domains is translated to the case of boundary conditions of the third kind.

In many calculations of temperature distributions one is confronted with bodies of complex configuration, and their direct analysis is rather complicated. Of enormous practical significance are investigations of temperature fields in composite and layered bodies such as the pipe fittings of liquid-metal systems with structural members of complex geometry [1]. The existing thermal calculations of complex bodies are carried out by the one-dimensional approximation with the introduction of an effective thermal conductivity. The possible domain of application of the one-dimensional solutions and the error have only been estimated for solid cylinders [2]. Similar estimates and calculations have essentially never been undertaken for double cylinders. However, the exact solutions for double cylinders are applicable only for a limited category of problems involving boundary conditions of the first and second kind. One-dimensional theories can be used for a double cylinder if one dimension is preferential over the others for the element investigated. If the one-dimensional approximation does not have sufficient accuracy, a multidimensional problem is formulated on the basis of information from the one-dimensional approximation. The design of piping systems is fraught with practical situations in which it is acceptable to limit the problem to small corrections to the one-dimensional theory without resorting to solution of the complete problem.

The method of boundary-layer corrections has been applied [3] to a boundary-value problem for a second-order elliptic equation in a thin domain with boundary conditions of the second kind on the surface. An analysis of the indicated method shows that it can also be applied to problems involving conditions of the third kind for a sufficiently small coefficient of the sought function. The solution is found in two iterative processes; in the case of conditions of the third kind the criterion for solvability of problems of one of the processes turns out to be the equation of the corresponding one-dimensional theory, and the subsequent iterations make it possible to find successive corrections to the one-dimensional solution. Below we give a solution of the heat-conduction for a two-layered cylinder by the method of boundary-layer corrections.

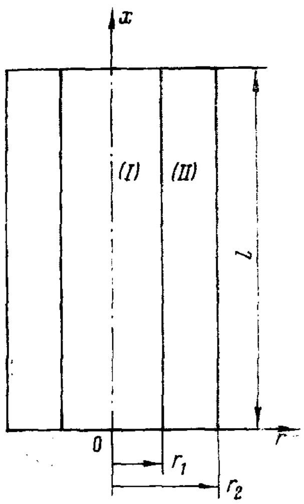

Fig. 1. Schematic of the problem.

We consider the steady-state temperature distribution in a double cylinder of radius r_2 and length l with a constant heat-transfer coefficient α on the lateral surface. The cylinder is composed of two coaxial sections with different heat conductivities λ (see Fig. 1). The lower end of the double cylinder is maintained at a constant temperature T_0.

The dimensionless excess temperature ϑ obeys the equation

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\vartheta}{\partial r}\right)+\frac{\partial^2\vartheta}{\partial x^2}=0\quad\left(\begin{array}{l}0<r<r_2\\0<x<l\end{array}\right) \tag{1}$$

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and the boundary conditions

$$\vartheta|_{r=0} \text{ finite; } -\lambda_2 \frac{\partial \vartheta}{\partial r}\bigg|_{r=r_2} = \alpha\vartheta|_{r=r_2}; \ \vartheta|_{x=0} = 1. \tag{2}$$

If we assume that

$$\vartheta = \begin{cases} \vartheta_1, & 0 < r < r_1, \\ \vartheta_2, & r_1 < r < r_2, \end{cases}$$

the problem can be reduced to the solution of two equations for domains I and II:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \vartheta_i}{\partial r}\right) + \frac{\partial^2 \vartheta_i}{\partial x^2} = 0, \ i = 1, 2 \tag{3}$$

with conditions for matching of the temperature and heat flux at the interface of the two cylinders:

$$\vartheta_1|_{r=r_1} = \vartheta_2|_{r=r_1}, \ \lambda_1 \frac{\partial \vartheta_1}{\partial r}\bigg|_{r=r_1} = \lambda_2 \frac{\partial \vartheta_2}{\partial r}\bigg|_{r=r_1}. \tag{4}$$

Various heat-conduction problems can be formulated for the double cylinder, depending on the boundary condition at the end surface $x = l$.

1. Let it be supposed that an arbitrary temperature distribution is given at the upper end:

$$\vartheta|_{x=l} = f(r). \tag{5}$$

A solution of problem (2)-(5) can be found by the separation of variables [5]:

$$\vartheta(r, x) = \sum_{n=1}^{\infty}\left[\sinh \mu_n \frac{x}{r_2} \times \int_0^{r_2} q(r) f(r) R_n(r)\, dr + \sinh \mu_n \frac{l-x}{r_2} \times \int_0^{r_2} q(r) R_n(r)\, dr\right] \frac{R_n(r)}{\sinh \mu_n \frac{l}{r_2} \int_0^{r_2} q(r) R_n^2(r)\, dr}, \tag{6}$$

where μ_n represents the positive roots of the transcendental equation

$$\frac{\mu}{\mathrm{Bi}} = \{[k_\lambda J_1(k_a\mu) Y_0(k_a\mu) - J_0(k_a\mu) Y_1(k_a\mu)] J_0(\mu) - J_0(k_a\mu) J_1(k_a\mu)(k_\lambda - 1) Y_0(\mu)\} / \{[k_\lambda J_1(k_a\mu) Y_0(k_a\mu) - J_0(k_a\mu) Y_1(k_a\mu)] J_1(\mu) - J_0(k_a\mu) J_1(k_a\mu)(k_\lambda - 1) Y_1(\mu)\} \tag{7}$$

and we have put $k_\lambda = \lambda_1/\lambda_2$ and $k_a = r_1/r_2$. The eigenfunctions of the problem

$$R_n(r) = \begin{cases} [J_0(k_a\mu_n) - \omega_n Y_0(k_a\mu_n)] J_0\left(\mu_n \frac{r}{r_2}\right), & 0 < r < r_1, \\ J_0(k_a\mu_n)\left[J_0\left(\mu_n \frac{r}{r_2}\right) - \omega_n Y_0\left(\mu_n \frac{r}{r_2}\right)\right], & r_1 < r < r_2, \end{cases} \tag{8}$$

where

$$\omega_n = \frac{\mathrm{Bi}\, J_0(\mu_n) - \mu_n J_1(\mu_n)}{\mathrm{Bi}\, Y_0(\mu_n) - \mu_n Y_1(\mu_n)}$$

are orthogonal on the interval $[0, r_2]$ with weight

$$q(r) = \begin{cases} rk_\lambda, & 0 < r < r_1, \\ r, & r_1 < r < r_2. \end{cases} \tag{9}$$

The given solution enables us to calculate the temperature distribution in a composite cylinder when the temperature distribution is specified in each stage. This fact permits the use of an iterative procedure for the calculation of the temperatures in a composite double cylinder.

2. Let the upper end be thermally insulated. Now condition (5) is is superseded by the homogeneous boundary condition of the second kind

$$\left.\frac{\partial \vartheta}{\partial x}\right|_{x=l} = 0. \tag{10}$$

We find the solution by separation of variables:

$$\vartheta(r,\ x) = \sum_{n=1}^{\infty} \frac{\cosh \mu_n \dfrac{l-x}{r_2}}{\cosh \mu_n \dfrac{l}{r_2}} \, \frac{\int\limits_0^{r_2} q(r)\, R_n(r)\, dr}{\int\limits_0^{r_2} q(r)\, R_n^2(r)\, dr}\, R_n(r), \tag{11}$$

where μ_n is determined from the transcendental equation (7) and the eigenfunctions $R_n(r)$ are evaluated from relation (8).

Under the condition $Bi \to 0$ the series converges so rapidly that only its first term needs to be retained. The coefficient μ_1 is determined from the transcendental equation (7) by the perturbation method. A small value of the criterion Bi corresponds to natural convective heat transfer in metals. The resulting solution can be used for the analysis of a low-temperature piping system with the use of a thermal insulating material.

3. The most interesting case occurs when convective heat transfer takes place at the upper end:

$$\left.\frac{\partial \vartheta}{\partial x}\right|_{x=l} = \begin{cases} -\dfrac{\alpha}{\lambda_1} \vartheta\Big|_{x=l}, & 0 < r < r_1, \\ -\dfrac{\alpha}{\lambda_2} \vartheta\Big|_{x=l}, & r_1 < r < r_2. \end{cases} \tag{12}$$

The cylinder is assumed to be slender, i.e., the ratio of the radius r_2 to the length l of the cylinder is considered to be a small parameter ε. We find an asymptotic solution of the problem for small ε. Following Dzhavadov [3], We represent the solution ϑ in the sum form

$$\vartheta = w + v, \tag{13}$$

where v is a function of the boundary-layer type.

For the function w Eq. (1) assumes the following form with the introduction of dimensionless coordinates $\bar{r} = r/r_2$, $\bar{x} = x/l$:

$$\frac{1}{\bar{r}} \frac{\partial}{\partial \bar{r}} \left(\bar{r} \frac{\partial w}{\partial \bar{r}} \right) + \varepsilon^2 \frac{\partial^2 w}{\partial \bar{x}^2} = 0. \tag{14}$$

Now the boundary conditions (2), (4), and (12) are rewritten

$$\begin{gathered} w|_{\bar{r}=0} \ \text{finite}, \quad \left.\frac{\partial w}{\partial \bar{r}}\right|_{\bar{r}=1} = -A\varepsilon^2 w|_{\bar{r}=1}; \\ w|_{\bar{x}=0} = 1, \quad \left.\frac{\partial w}{\partial \bar{x}}\right|_{\bar{x}=1} = \begin{cases} -\dfrac{A\varepsilon}{k_\lambda} w\Big|_{\bar{x}=1}, & 0 < \bar{r} < k_a; \\ -A\varepsilon w|_{\bar{x}=1}, & k_a < \bar{r} < 1; \end{cases} \\ w|_{\bar{r}=k_a-0} = w|_{\bar{r}=k_a+0}, \quad k_\lambda \left.\frac{\partial w}{\partial \bar{r}}\right|_{\bar{r}=k_a-0} = \left.\frac{\partial w}{\partial \bar{r}}\right|_{\bar{r}=k_a+0}, \end{gathered} \tag{15}$$

where $A\varepsilon^2 = r_2\alpha/\lambda_2 = Bi$; $A\varepsilon = \alpha l/\lambda_2$.

To find the function v we introduce the dimensionless coordinates

$$\rho = \frac{r}{r_2}; \quad \zeta = \frac{l-x}{r_2}. \tag{16}$$

Then Eq. (1) is rewritten

$$\frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho \frac{\partial v}{\partial \rho} \right) + \frac{\partial^2 v}{\partial \zeta^2} = 0 \ \left(\begin{matrix} 0 < \rho < 1 \\ 0 < \zeta < \infty \end{matrix} \right), \tag{17}$$

and the boundary conditions acquire the form

$$\varepsilon \frac{\partial w}{\partial \bar{x}}\Big|_{\bar{x}=1} - \frac{\partial v}{\partial \zeta}\Big|_{\zeta=0}$$

$$= \begin{cases} -\dfrac{A\varepsilon^2}{k_\lambda}\left(w|_{\bar{x}=1} + v|_{\zeta=0}\right), & 0 < \bar{r},\ \rho < k_a \\ -A\varepsilon^2\left(w|_{\bar{x}=1} + v|_{\zeta=0}\right), & k_a < \bar{r},\ \rho < 1; \end{cases}$$

$$\frac{\partial v}{\partial \rho}\Big|_{\rho=1} = -A\varepsilon^2 v|_{\rho=1},\quad v|_{\rho=0} \text{ finite};$$

$$v|_{\rho=k_a-0} = v|_{\rho=k_a+0},\quad k_\lambda \frac{\partial v}{\partial \rho}\Big|_{\rho=k_a-0} = \frac{\partial v}{\partial \rho}\Big|_{\rho=k_a+0}. \tag{18}$$

The functions w and v are evaluated by different iterative processes. We refer to the process used to find w as the outer expansion, and to the process for v as the inner expansion.

We begin the solution of the problem by finding the outer expansion. We represent the function w in the form

$$w = w_0 + \varepsilon w_1 + \varepsilon^2 w_2 + \dots . \tag{19}$$

Substituting the series (19) into Eq. (14) and the boundary conditions (15) and equating coefficients of like powers of ε, we arrive at the following problem for the determination of w_0 (we drop the bars over the dimensionless variables):

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w_0}{\partial r}\right) = 0,$$

$$w_0|_{r=0} \text{ finite},\quad \frac{\partial w_0}{\partial r}\Big|_{r=1} = 0;$$

$$w_0|_{r=k_a-0} = w_0|_{r=k_a+0},\quad k_\lambda \frac{\partial w_0}{\partial r}\Big|_{r=k_a-0} = \frac{\partial w_0}{\partial r}\Big|_{r=k_a+0}. \tag{20}$$

After integration we have

$$w_0 = \bar{w}_0(x). \tag{21}$$

For the function w_1, analogously, we obtain the following dependence on the longitudinal coordinate:

$$w_1 = \bar{w}_1(x). \tag{22}$$

Now for the third term w_2 of the outer expansion the problem assumes the form

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w_2}{\partial r}\right) = -\frac{d^2\bar{w}_0}{dx^2}, \tag{23}$$

$$w_2|_{r=0} \text{ finite},\quad \frac{\partial w_2}{\partial r}\Big|_{r=1} = -A\bar{w}_0|_{r=1};$$

$$w_2|_{r=k_a-0} = w_2|_{r=k_a+0},\quad k_\lambda \frac{\partial w_2}{\partial r}\Big|_{r=k_a-0} = \frac{\partial w_2}{\partial r}\Big|_{r=k_a+0}. \tag{24}$$

The integration of Eq. (23) subject to conditions (24) yields

$$w_2 = \begin{cases} -\dfrac{d^2\bar{w}_0}{dx^2}\dfrac{r^2}{4} + \bar{w}_2(x), & 0 < r < k_a, \\ -\dfrac{d^2\bar{w}_0}{dx^2}\dfrac{r^2}{4} + C_1 \ln\dfrac{r}{k_a} + \bar{w}_2(x), & k_a < r < 1, \end{cases} \tag{25}$$

where

$$C_1 = \frac{k_a^2}{2}(1 - k_\lambda)\frac{d^2\bar{w}_0}{dx^2}.$$

The condition for solvability of problem (23)-(24) leads to the following equation for the determination of $\bar{w}_0$:

$$\frac{d^2\bar{w}_0}{dx^2} - m^2\bar{w}_0 = 0, \quad (26)$$

where $m^2 = 2A/1-(1-k_\lambda)k_a^2$, because $[1-(1-k_\lambda)k_a^2] > 0$ and $A > 0$.

Let $w_0(x)$ satisfy the following conditions with respect to x:

$$\bar{w}_0 |_{x=0} = 1, \quad \left.\frac{d\bar{w}_0}{dx}\right|_{x=1} = 0. \quad (27)$$

The solution of this problem is given by the relation

$$\bar{w}_0(x) = \frac{\cosh m(1-x)}{\cosh m}. \quad (28)$$

The given solution corresponds to the temperature distribution in a rod with a thermally insulated end surface [6]. It follows therefrom that for a small value of the Biot criterion the one-dimensional approximation can be used for a double rod. An analogous result is obtained from (11) for the limiting situation $Bi \to 0$.

For the function $\bar{w}_1(x)$ we obtain an equation of the type (26) with the boundary conditions

$$\bar{w}_1 |_{x=0} = 0; \quad \left.\frac{d\bar{w}_1}{dx}\right|_{x=1} = q_1, \quad (29)$$

where q_1 is to be determined later.

We now consider the iterative process for the function v. We represent v in the form

$$v = \varepsilon^2 v_0 + \varepsilon^3 v_1 + \ldots \quad (30)$$

The equation for v_0 takes the form

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial v_0}{\partial\rho}\right) + \frac{\partial^2 v_0}{\partial\zeta^2} = 0 \quad \left(\begin{matrix} 0<\rho<1 \\ 0<\zeta<\infty \end{matrix}\right), \quad (31)$$

and the boundary conditions are rewritten

$$\left.\frac{\partial v_0}{\partial\zeta}\right|_{\zeta=0} = q_1 + \begin{cases} \frac{A}{k_\lambda}\bar{w}_0(1), & 0<\rho<k_a \\ A\bar{w}_0(1), & k_a<\rho<1; \end{cases}$$

$$v_0|_{\rho=0} \text{ finite}, \quad v_0|_{\zeta\to\infty} \to 0, \quad \left.\frac{\partial v_0}{\partial\rho}\right|_{\rho=1} = 0;$$

$$v_0|_{\rho=k_a-0} = v_0|_{\rho=k_a+0}, \quad k_\lambda \left.\frac{\partial v_0}{\partial\rho}\right|_{\rho=k_a-0} = \left.\frac{\partial v_0}{\partial\rho}\right|_{\rho=k_a+0}. \quad (32)$$

The eigenfunctions of this problem are orthogonal on the interval [0, 1] with weight

$$r(\rho) = \begin{cases} \rho k_\lambda, & 0<\rho<k_a, \\ \rho, & k_a<\rho<1. \end{cases}$$

The condition for solvability of problem (31)-(32) yields

$$\int_0^{k_a} \rho k_\lambda q_1 d\rho + \int_{k_a}^1 \rho q_1 d\rho + \int_0^{k_a} \frac{A}{k_\lambda}\bar{w}_0(1)\rho k_\lambda d\rho + \int_{k_a}^1 A\bar{w}_0(1)\rho d\rho = 0, \quad (33)$$

whence

$$q_1 = -\frac{m^2}{2}\bar{w}_0(1) = -\frac{m^2}{2\cosh m}.$$

The solution for $\bar{w}_1(x)$ can be represented in explicit form by means of the value found for q_1:

$$\bar{w}_1(x) = -\frac{m}{2\cosh^2 m}\sinh mx. \quad (34)$$

The function $v_0(\rho, \zeta)$ is determined from Eq. (31) and the boundary conditions (32) by separation of variables:

$$v_0(\rho,\ \zeta) = \frac{k_a(1-k_\lambda)\left(\frac{m^2}{2}-A\right)}{\cosh m} \sum_{n=1}^{\infty} \frac{J_1(k_a\mu_n)}{\mu_n^3 N_n} R_n(r)\, e^{-\mu_n\zeta}, \tag{35}$$

where

$$R_n(r) = \begin{cases} R_{1n} = J_0(\mu_n\rho), & 0<\rho<k_a \\ R_{2n} = \dfrac{J_0(\mu_n\rho)\, Y_1(\mu_n) - J_1(\mu_n)\, Y_0(\mu_n\rho)}{J_0(\mu_n k_a)\, Y_1(\mu_n) - J_1(\mu_n)\, Y_0(\mu_n k_a)} J_0(\mu_n k_a), & k_a<\rho<1; \end{cases}$$

$$N_n = \int_0^1 r(\rho) R_n^2(\rho)\, d\rho = \frac{1}{2\mu_n^2}\Big\{ k_a^2 k_\lambda (1-k_\lambda) J_1^2(\mu_n k_a) + \mu_n^2 k_a^2 J_0^2(\mu_n k_a)(k_\lambda - 1) + \frac{4}{\pi^2} \frac{J_0^2(\mu_n k_a)}{[J_0(\mu_n k_a)\, Y_1(\mu_n) - J_1(\mu_n)\, Y_0(\mu_n k_a)]^2}\Big\}$$

and μ_n represents the positive roots of the transcendental equation

$$[k_\lambda J_1(k_a\mu)\, Y_0(k_a\mu) - J_0(k_a\mu)\, Y_1(k_a\mu)]\, J_1(\mu) - J_0(k_a\mu)\, J_1(k_a\mu)(k_\lambda - 1)\, Y_1(\mu) = 0. \tag{36}$$

To determine the function w_2 we need to analyze the following problem for the function w_4:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w_4}{\partial r}\right) = -\frac{\partial^2 w_2}{\partial x^2},$$

$$w_4|_{r=0}\ \text{finite},\quad \left.\frac{\partial w_4}{\partial r}\right|_{r=1} = -A w_2|_{r=1}; \tag{37}$$

$$w_4|_{r=k_a-0} = w_4|_{r=k_a+0},\quad k_\lambda \left.\frac{\partial w_4}{\partial r}\right|_{r=k_a-0} = \left.\frac{\partial w_4}{\partial r}\right|_{r=k_a+0}. \tag{38}$$

Taking Eqs. (25) into account, we rewrite Eq. (37) as follows:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w_4}{\partial r}\right) = \begin{cases} \dfrac{d^4\bar{w}_0}{dx^4}\dfrac{r^2}{4} - \dfrac{d^2\bar{w}_2}{dx^2}, & 0<r<k_a, \\ \dfrac{d^4\bar{w}_0}{dx^4}\dfrac{r^2}{4} - (1-k_\lambda)\dfrac{k_a^2}{2}\ln\dfrac{r}{k_a}\dfrac{d^4\bar{w}_0}{dx^4} - \dfrac{d^2\bar{w}_2}{dx^2}, & k_a<r<1. \end{cases} \tag{39}$$

Integrating (39), we obtain

$$w_4 = \begin{cases} \dfrac{d^4\bar{w}_0}{dx^4}\dfrac{r^4}{64} - \dfrac{d^2\bar{w}_2}{dx^2}\dfrac{r^2}{4} + D\ln r + \bar{w}_4(x), & 0<r<k_a, \\ \dfrac{d^4\bar{w}_0}{dx^4}\dfrac{r^4}{64} - (1-k_\lambda)\dfrac{k_a^2}{2}\dfrac{d^4\bar{w}_0}{dx^4}\dfrac{r^2}{2}\left(\ln\dfrac{r}{k_a} - 1\right) \\ \quad - \dfrac{d^2\bar{w}_2}{dx^2}\dfrac{r^2}{4} + D_1\ln r + \bar{w}_4^*(x), & k_a<r<1. \end{cases} \tag{40}$$

The temperature and heat-flux matching conditions stipulate the following relations between the constants:

$$\bar{w}_4^*(x) = \bar{w}_4(x) - (1-k_\lambda)\frac{k_a^4}{8}\frac{d^4\bar{w}_0}{dx^4} - D_1\ln k_a;$$

$$D_1 = -\frac{d^2\bar{w}_2}{dx^2}\frac{k_a^2}{2}(k_\lambda - 1) + \frac{3}{16}k_a^4(k_\lambda - 1)\frac{d^4\bar{w}_0}{dx^4}. \tag{41}$$

The condition for solvability of the problem for w_4 permits us to derive a differential equation for the evaluation of the function $\bar{w}_2$:

$$\frac{d^2\bar{w}_2}{dx^2} - m^2\bar{w}_2 = E\cosh m(1-x), \tag{42}$$

where

$$E = \frac{m^2}{\cosh m}\frac{m^2}{A}\left[\frac{m^2}{16} + m^2(1-k_\lambda)\frac{k_a^2}{4}\left(\ln k_a + \frac{1}{2}\right) + \frac{3m^2}{16}k_a^4(k_\lambda - 1) - \frac{A}{4} - A\ln k_a\frac{k_a^2}{2}(1-k_\lambda)\right],$$

subject to the boundary conditions

$$\bar{w}_2|_{x=0} = 0, \quad \left.\frac{d\bar{w}_2}{dx}\right|_{x=l} = q_2. \tag{43}$$

The dimensionless heat flux q_2 has to be determined from the condition for solvability of the problem for the function v_1.

The function $v_1(\rho, \zeta)$ satisfies (31) and the boundary conditions

$$\left.\frac{\partial v_1}{\partial \zeta}\right|_{\zeta=0} = q_2 - \begin{cases} \dfrac{A}{k_\lambda}\dfrac{m}{2\cosh^2 m}\sinh m, & 0 < r < k_a, \\ A\dfrac{m}{2\cosh^2 m}\sinh m, & k_a < r < 1; \end{cases}$$

$$v_1|_{\zeta\to\infty} \text{ finite}, \quad \left.\frac{\partial v_1}{\partial \rho}\right|_{\rho=1} = 0, \quad v_1|_{\rho=0} \text{ finite},$$

$$v_1|_{\rho=k_a-0} = v_1|_{\rho=k_a+0}, \quad k_\lambda\left.\frac{\partial v_1}{\partial \rho}\right|_{\rho=k_a-0} = \left.\frac{\partial v_1}{\partial \rho}\right|_{\rho=k_a+0}. \tag{44}$$

the condition for solvability of the problem for v_1

$$\int_0^{k_a}\rho k_\lambda q_2 d\rho + \int_{k_a}^{1}\rho q_2 d\rho - \int_0^{k_a}k_\lambda\rho\frac{A}{k_\lambda}\frac{m\sinh m}{2\cosh^2 m}d\rho - \int_{k_a}^{1}\rho A\frac{m\sinh m}{2\cosh^2 m}d\rho = 0$$

determines the value of the dimensionless heat flux q_2:

$$q_2 = \frac{1}{4}m^3\frac{\sinh m}{\cosh^2 m}. \tag{45}$$

Consequently, problem (42)-(43) can now be solved in explicit form for the determination of $\bar{w}_2$:

$$\bar{w}_2 = \frac{1}{m\cosh m}\left(\frac{1}{4}m^3\frac{\sinh m}{\cosh^2 m} - \frac{1}{2}E\right)\sinh mx - \frac{E}{2m}x\sinh m(1-x). \tag{46}$$

The next-higher terms of the expansions (19) and (30) can be computed analogously, where the boundary conditions for the functions w_{i+1} are determined by the condition for solvability of the corresponding problems for the functions v_i.

The final solution can be extended to small values of the parameter ε. The given solution has the singular feature that the dependence on the radius is felt only in the second-order terms in ε. The problem can therefore be regarded as one-dimensional with a small correction for the radial variation of the fundamental parameters. The proposed analytical method can also be extended to other cases of boundary conditions. In particular, it can be used to solve the problem of the steady-state temperature distribution in a double cylinder with nonideal contact between the inner and outer cylinders.

NOTATION

T	is the absolute temperature;
x, r	are the cylindrical coordinates;
$\vartheta = (T-T_L)/(T_0-T_L)$	is the dimensionless excess temperature;
$Bi = \alpha r_2/\lambda$	is the Biot number;
α	is the heat-transfer coefficient;
λ	is the thermal conductivity;
r_i	is the radius of the i-th cylinder;
l	is the height of the cylinder;

$q(r)$	is a weighting function;
$J_0(z)$, $J_1(z)$, $Y_0(z)$, $Y_1(z)$	are Bessel functions of the first and second kind;
$\varepsilon = r_2/l$	is the small parameter of the problem;
$k_r = r_1/r_2$, $k_\lambda = \lambda_1/\lambda_2$	are dimensionless coefficients.

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